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THE DEVELOPMENT OF SERIAL COMPLETION STRATEGIES: AN INFORMATION PROCESSING ANALYSIS

By DAVID KLAHR
Department of Psychology, Carnegie-Mellon University, Pittsburgh, Pennsylvania, U.S.A.

AND JOHN G. WALLACE
Department of Education, University of Warwick

An information processing model of a complex task involving several important Piagetian concepts is introduced and then extended to a more difficult class of problems often used to test adult cognitive abilities. In comparing this extended model both to human performance and to the process model by Simon & Kotovsky, the criterion of developmental tractability is introduced. A detailed study of a child's performance during a crucial transition from one stage of problem complexity to another indicates that a mixture of both models seems to provide the best explanation of problem-solving behaviour.

The ability to detect environmental regularities is a cognitive skill essential for survival. Man has a propensity to seek and capacity to find serial patterns in such diverse areas as music, economics, and the weather. Even when no true pattern exists, humans attempt to construct one that will enable them to predict the sequence of future events (Feldman *et al.*, 1963). An understanding of these abilities has been facilitated through the development of computer simulation models. These models have postulated a sufficient set of basic processes, which, when appropriately organized, enable humans to detect serial pattern. The most widely known example of such an information processing theory is by Simon & Kotovsky (1963).

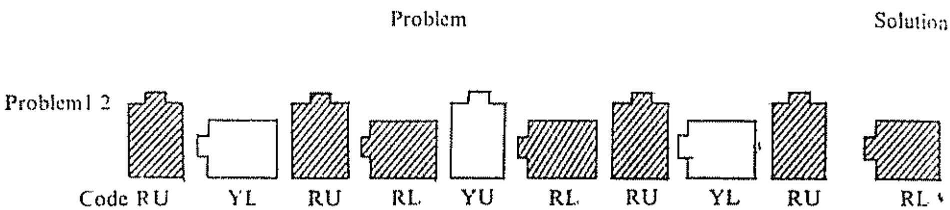
The study of the components of serial pattern detection has also been the object of much of the Piagetian research into the development of cognitive skills. The ability of a child to solve series completion problems of the kind shown in Fig. 1 requires the mastery of several skills that are believed to be distinguishing characteristics of children who have attained the Piagetian level of concrete operations. Such problems tap the child's facility with set and relational concepts, two-dimensional transformations, multiplication of sets and relations and serial anticipation (Inhelder & Piaget, 1964).

An information processing approach to the study of cognitive development may prove mutually beneficial to both areas of psychology. This paper attempts an integration of the two fields through an information processing analysis of two types of series completion problems. Two different information processing models are presented, and it is shown that one model can be considered to be the developmental precursor of the other. The evidence presented in support of this position is taken from a year-long longitudinal study of 5- to 6-year-old children solving, among other things, over 300 series completion problems.

Colour-orientation series completion problems

The problem in Fig. 1 is presented to a 5-year-old child and he is asked 'what comes next?' The solution can be obtained by considering the dimensions of colour and orientation separately. The colour problem is a repeated RYR; the colour solution is Red. The orientation problem is a repeated UL; the solution is Left. The composite solution is Red Left.

As in any series completion problem, there is no absolutely right answer, but children aged 5–7 can solve a variety of problems of this kind (see Table 1), producing solutions and descriptions of their solution process that agree with the 'separation of attributes' strategy described above. Some problems are particularly difficult



Letter code: 1st letter colour: Red, Yellow, Green, Blue
2nd letter orientation: Right, Up, Left, Down

Fig. 1. A two-attribute series completion problem.

Table 1. *Test series of colour-orientation problems*

(Colour: *B*lue, *G*reen, *R*ed, *Y*ellow. Orientation: *D*own, *L*eft, *R*ight, *U*p.)

No.	Problem	Solution
1	BL YD BL YD BL	YD
2	BU YU BR YR BU YU	BR
3	BR BR YD BR BR YD	BR
4	BU YD YU BD YU YD	BU
5	BU BU YL YL BU BU YL YL	BU
6	BL YD BL GU YD GU BL YD BL	GU
7	BD YD BD GR RD GR BD YD BD	GR
8	BD YD BD YR BR YR BD YD BD	YR
9	BU BU BR YD YD YR BU BU BR	YD
10	BU BR BU BD BR BD BU BR BU	BD
11	BR YR YD BL YL YD BR YR YD	BL
12	BU YU BR YD BD YR BU YU BR	YD
13	BD YR GR BD YR GR	BD
14	BR YR GD BR YR GD	BR
15	BR YD GL BR YD GL	BR
16	BR YU GR BU YR	GU
17	BR GU YR RU GR	YU
18	BR YU RR BU YR	RU
19	YD GL BD YL GD	BL
20	RU GD YU RD GU	YD
21	GU BR YU GR	BU
22	RU GD YU RD	GU
23	RR YL GR RL	YR
24	RD BU GD RU	BD

requiring the production of a colour-orientation combination that does not appear in the series, e.g. RR YU GR RU YR. The colour pattern is RYG: the colour solution is G. The orientation pattern is RU: the orientation solution is U. Therefore the composite solution is GU, an object that does not occur in the problem.

Proposed solution strategy

What capacities must the child possess in order to be successful on these tests? The evidence suggests that some subjects solve these problems by constructing templates of increasing size until they find a recurring pattern. Consider the single attribute problem shown in line 1 of Fig. 2. (Assume the problem consists of coloured tiles in the sequence: Red Green Red Red Green Red Red ——.)

```
1        R G R R G R R –        problem
2        R                      initialize
3          R                    match
4        R G                    initialize
5            R G                match
6        R G R                  initialize
7              R G R            match
8                    R G R      match
9                      G        produce
```

Fig. 2. Template building procedure for a single attribute.

```
     Problem 1.2.     RU YL RU RL YU RL RU YL RU –

           Colour                        Orientation

R Y R R Y R R Y R –              U L U L U L U L U
R                                U
  R                                U
R Y                              U L
    R Y                              U L
R Y R                                    U L
      R Y R                                  U L
            R Y R                                U L
           (R)                                   (L)
                   (RL)
```

Fig. 3. Template building procedure on two-attribute objects. Separation of attributes.

The strategy initially assumes the pattern is R (line 2). This pattern is constructed and matched for identity against the next contiguous part of the problem (line 3). The match fails, since R is not the same as G, and the pattern is reinitialized to RG (line 4). This is matched unsuccessfully against RR (line 5) and a second reinitialization constructs a pattern of RGR (line 6). When matched against the adjacent part of the problem (line 7) it is successful, hence the pattern moves on to make another match (line 8). In the process of making this match, however, the problem list is exhausted, and the corresponding symbol in the pattern is produced (line 9).

The basic processes required in this strategy are the ability to recognize 'sames' when they occur and the ability to keep track of position within two lists: the pattern and the problem.

The extension of this strategy to two-attribute problems is quite straightforward: it requires only the ability to treat the attributes independently, and to remember the correct value of the first attribute while working on the second. Fig. 3 shows an example of a two-attribute problem being solved independently on each attribute. In Fig. 4 the same problem is solved without separating attributes, i.e. by treating each colour-orientation combination as a distinct object. This latter approach produces rather large pattern templates and it will not work on certain classes of problems.

Problem 1.2. RU YL BU RL YU RL RU YL RU –

```
RU
    RU
RU YL
        RU YL
RU YL RU
            RU YL RU
RU YL RU RL
                RU YL RU RL
RU YL RU RL YU
                    RU RL RU RL YU
RU YL RU RL YU RL
                        RU YL RU RL YU RL
                                  (RL)
```

Fig. 4. Template building procedure on two attribute objects. No separation of attributes. (Wholistic.)

Evidence supporting template strategy

This section presents some examples of the kind of data upon which the model is based. Some of it is consistent with the hypothesis that the template model is a reasonable description of the subject's problem-solving processes.

A few segments are presented of the verbal protocol of a 5-year-old child solving series completion problems similar to those described above. The problems are presented to the subject and he is asked to state and justify his prediction. The experimenter responds in a manner designed to extract from the subject information about the problem solving procedure.

Some evidence for the use of a template is present in the protocol for the following problem: BU BR BU BD BR BD BU BR BU. The protocol is:

'*Blue down [points to fourth object in series].* Why? *These 3 [1,2,3] are the same as these [7,8,9] so, uh, if a blue one facing downwards went there it would be fair 'cause then these four [1,2,3,4] would be the same as these four [7,8,9 and choice].*'

Notice that the subject points to the part of the problem that corresponds to the part of the template from which the answer will be produced. His final explanation further supports the notion of a repeated template.

There is no evidence in the above protocol of an independent treatment of colour and orientation. The next problem supports the hypothesis that the subject has the ability to consider attributes separately. However, it does not support the template strategy. It is included here to provide an example of a quite different strategy that is sufficiently powerful to solve many of the problems. Problem: RR GU YR RU GR. Protocol:

'*Yellow up.* Why? *Well there's two of the ones upwards and three of the ones facing across and if that other one went there slanting up there they would all have a partner. 'Cause the red's got a partner, the green's got a partner but the yellow hasn't. So it's a yellow one facing up, there's two of these facing upwards.*'

Here the subject has produced a correct solution by attending first to orientation and then to colour. However, he has treated the problem as an unordered collection of objects, rather than as a series of objects. If in fact this is his conceptualization of the problem, then it is not at all clear that he could indefinitely extrapolate this series.

The 'fairness' explanation is used by this subject for many of the problems. (Occasionally objects get 'partners' because 'they are lonesome'.) After a series of problems such as the one above, in which a fairness strategy produces a unique and correct solution, the subject is presented with a problem of the following type: RU GD YU RD. Fairness alone is insufficient here, because both the green and yellow objects are lonesome. The subject seems to have used a combination of a colour template and a fairness approach to these problems:

'*A green up.* Why? *Well it goes in order. The red one's got a partner, the green one hasn't—did I say a green facing up one?* Yes. *The red one's got a partner, the green one hasn't; the green one's getting one. Then there'll be a yellow one facing down, then it'll be one, two, three [indicates objects 2,4.6 all down] and one, two, three [indicates objects 1,3,5 all up].*'

One can analyse the full set of protocols from which these samples have been taken in an attempt to distil a set of 'pure' strategies. From the varied verbal reports and performances of the subjects it has become clear that the subjects are highly idiosyncratic in their use of strategies. An intensive analysis of individual subjects has yielded a small set of strategies that seem to be employed by subjects in various mixes. Two such strategies have already been described. (See Figs. 3–4.) Two other common ones are: (1) a backward scanning strategy in which a template of the last few objects in the series is matched against the problem; (2) a 'fairness' strategy which seeks an equal number of colours for each orientation or an equal number of orientations for each colour. (See the last protocol above.)

Each of these strategies can be described in completely unambiguous form, i.e. in a computer programming language, and then applied to the same series of problems presented to our subjects. (See Table 1.)

The results for four strategies and a single subject are shown in Table 2. They indicate, for those cases in which an unambiguous solution exists, what that solution is and whether or not it is correct. This provides a base-line against which to interpret a pattern of choices by the subject. If only the subject's *choices* are used, then the combination of strategies that was consistent with the choice pattern could be sought. However, by also using the protocol in the manner indicated earlier in this section, one can determine which strategies, if they were being used, would be consistent with the protocols. The results of such an analysis of the protocols are also listed in Table 2. For the first 15 problems this subject produces solutions and protocols that are consistent with both the 'wholistic' template strategy (S2) and a 'fairness' strategy (S4). For the final problems he is forced to switch to the separate treatment of dimensions embodied in the 'colour-orientation template' strategy (S1) in conjunction with S4.

This then is one kind of analysis that can be performed. Extensions to other subjects, the same subject over time, and other problems can all be treated at the same level of precision. One can also descend a level to study the relationship between the postulated basic processes underlying the proposed strategies and the more conventional cognitive abilities. In the remainder of this paper we limit ourselves to an extension of a different type: a change in the problem difficulty

Table 2. *Solutions produced by each strategy, subject solution and inferred use of strategy by subjects*

(S 1: template colour-orientation; S 2: template wholistic; S 3: backward template; S 4: ran…

Problem	Solution produced					Inferred strategy
	S 1	S 2	S 3	S 4	Subject	
1	YD	YD	YD		YD	S 4
2	BR	BR	BR		BR	S 2, S 4
3	BR	BR	BR		BR	S 2
4	BU	BU	—		BR*, BU	S 3, S 2, S 4
5	BU	BU	BU		BU	S 2
6	GU	GU	GU		GU	S 2, S 4
7	GR	GR	GR		GR	S 2, S 4
8	YR	YR	YR		YR	S 2, S 4
9	YD	YD	YD		YD	S 2, S 4
10	BD	BD	BD		BD	S 2, S 4
11	BL	BL	BL		BL	S 2, S 4
12	YD	YD	YD		YD	S 2, S 4
13	BD	BD	BD		BD	S 2, S 4
14	BR	BR	BR		BR	S 2, S 4
15	BR	BR	BR		BR	S 2, S 4
16	GU	BR*	—	GU	GR*, GU	S 1, S 4
17	YU	RR*	—	YU	YU	S 4
18	RU	BR*	—	RU	RL*	S 4
19	BL	YD	—	BL	BR*	S 4
20	YD	RU*	—	YD	YD	S 4
21	BU	GU*	—	—	BU	S 1, S 4
22	GU	RU*	—	—	GU	S 1, S 4
23	YR	RR*	—	—	YL*	S 1, S 4
24	BD	RD*	—	—	BD	S 1, S 4

* Incorrect.

LETTER SERIES COMPLETION

Letter series completion (LSC) problems are widely used for testing human capacity to recognize and extrapolate serial patterns. A successful extrapolation of the problems in Table 3 implies that the subject has discovered the pattern underlying the series through a process of induction. This section describes a template building procedure quite similar to the first one proposed for a colour-orientation problem.

By building a model that works with a pattern of *relations* instead of simply with objects, it is possible to use the same template building procedure that works so well in the non-ordered problems. Consider the letter series completion problem shown in Fig. 5. Assume, as before, that one can build up a pattern from the left, increasing its size with each failure encountered. Start with a *symbol pattern* of the first item in the series, U (line 1), and match it against its neighbour to see if any of the following relations on the alphabet hold: prior, same, next, double next (represented by *p*, *s*, *n*, *t*). No relation is found between U and R (indicated by *o*); therefore return to the INIT phase, obtaining the symbol pattern UR (line 2). In the FIND phase (line 4 relation is found between U and T, but not between R and U, and again return to INIT, this time producing URT as the symbol pattern. When URT is compared with its neighbouring symbols UST, a successful *relational pattern* consisting of *sns* is obtained (line 6). Perhaps this relation will be found to hold throughout the remainder of the problem. First update the symbol pattern by applying the relational pattern to the symbol pattern, producing UST (line 7), then move the symbol pattern along and test to see if the relation between UST and UTT is *sns* (line 8). It is; therefore cycle through APPLY and TEST once more. Upon encountering the end of the problem apply the corresponding relation to the symbol and produce U (line 11).

```
       U R T U S T U T T U -
 1     U                                 init       1
         o
 2       U                               find       2
 3     U R                               init       3
           p o
 4         U R                           find       4
 5     U R T                             init       5
             s n s
 6           U R T                       find       6
 7           U S T                       apply      7
                   s n s
 8                 U S T                 test       8
 9                 U T T                 apply      9
                         s n s
10                       U T T           test      10
11                         U             produce   11

              series description: sns(URT)
```

Fig. 5. Construction of relational template.

The entire series can be described in terms of the initial values and the relational pattern. For the problem in Fig. 5 it is *sns*(URT). In this notation, the compound operator *sns* is applied sequentially to the initial compound argument URT, outputting URT and updating to UST. This cycle repeats indefinitely.

This model has been implemented in an information processing language, POP-2 (Burstall & Popplestone, 1968), and the resultant programme has been run on the problems listed in Table 3. Set I is taken from Simon & Kotovsky (1963), and Set II from an unpublished paper written in the psychology department at the University of Michigan by DuCharm *et al.* (1965). As indicated in Table 3 the model correctly solved all problems in Set I and all but six in Set II. If endowed with a test for 'triple next', then the model can solve all but one problem in Set II.

Comparison with the Simon & Kotovsky model

Simon & Kotovsky's theory of human acquisition of concepts for sequential patterns asserts that humans: (1) 'attain a serial pattern concept by generating and fixating a pattern description of that concept'; (2) 'have stored in memory a programme capable of interpreting and executing descriptions of serial patterns'; (3) 'have stored in memory a programme... capable of detecting relations and recording a pattern description for a simple sequence'.

At a general level the template model rests upon this same set of assertions.

However, after these ambiguous propositions have been translated into a well defined sequence of operations some important differences become apparent. Simon & Kotovsky note that any objection to the assertion that humans use patterns similar

Table 3. *Problems posed for letter series completion programme*

(s = same, n = next, p = prior, t = double next.)

	Problem	Correct solution	Programme output: Answer	Programme output: Description
Set I				
1	C D C D C D	C	C	ss
2	A A A B B B C C C D D	D	D	nnn
3	A T B A T A A T B A T	A	A	ssss†
4	A B M C D M E F M G H M	I	I	tts
5	D E F G E F G H F G H I	G	G	nnnn
6	Q X A P X B Q X A	P	P	sss†
7	A D U A C U A E U A B U A F U A	A	A	snssps
8	M A B M B C M C D M	D	D	snn
9	U R T U S T U T T U	U	U	sns
10	A B Y A B X A B W A B	V	V	ssp
11	R S C D S T D E T U E F	U	U	nnnn
12	N P A O Q A P R A Q S A	R	R	nns
13	W X A X Y B Y Z C Z A D A B	E	E	nnn
14	J K O R K L R S L M S T	M	M	nnnn
15	P O N O N M N M L M L K	L	L	ppp
Set II				
1	A A A B B B C C C D D	D	D	nn-
2	W X A X Y B Y Z C Z A D A B	E	E	nnn
3	M A B M B C M C D M	D	D	snn
4	N P A O Q A P R A O S A	R	R	nns
5	F G H H I J K K L M N	N	F	—*
6	J K Q R K L R S L M S T	M	M	nnnn
7	J K I L L M I N N O I P	P	P	ttst
8	D E F G E F G H F G H I	G	G	nnnn
9	E F G G H H I J J K K L M M	N	E	—*
10	A G B C H C I D E J E K F	G	G	ttttt
11	R R S T T U R V W W X R	Y	R	s*
12	N Q M R O P S M T Q R U M V	S	S	ttstt
13	Q R R S T T U V	V	V	ttt
14	E A F G B H I C J K D L	M	M	tnt
15	M R N O R P Q R R	S	S	tst
16	F G J G G K H G L I G	M	M	nsn
17	Q R S S T U V V W X	Y	Q	—*
18	A D B E C E D F E F F	G	G	tntn
19	P M Q R R M S T T M U V	V	V	tstt
20	G D S H I E S J K F S L M	G	G	tnst
21	K L L M N O P P Q R	S	K	—
22	A B G C H C D H E I E F I G J	G	G	ttntn
23	I J J U K L M M U N O P P	U	I	—*
24	C A T B D E C T D F G E T F H	I	I	ttstt

* These can be solved if the relation 'triple next' is included in the set of relations.
† Notice that these pattern descriptions are only partially complete

to their particular description of such patterns 'would be more convincing if it could be shown that the patterns could be described in a manner quite different from the one we have proposed'.

The Simon & Kotovsky descriptions refer to two 'slots' in immediate memory, M1 and M2, which are pointers to positions on ordered circular lists. Usually these lists are either the forward or backward alphabet, but occasionally the list is constructed *ad hoc* from the problem elements. The basic process of moving the pointer to the next position on the list is represented by the notation N( ). Thus for Problem 2 in Set I (Table 3), the pattern AAABBBCCCDD is described in the notation: M1 = Alph; A. M1, M1, M1, N(M1). They describe the interpretation as follows:

'We set a variable, M1, equal to the first letter, A, of the alphabet. Each period is executed by producing M1 three times, and then replacing M1 by the *next* (N) letter of the alphabet.'

Only one additional operator is required to enable them to describe all the problems in Table 3. The notation E(M1, M2) means that M1 is set to the current value of M2. Their description of Problem 15 is: M1 = M2 = Balph; P. M1, N(M1), M1, N(M1), M1, N(M2), E(M1, M2). For the template model the description is ppp (PON).

Differences in pattern description

The underlying logic of Simon & Kotovsky assumes that the subject is keeping track of his position(s) in the alphabet, whereas the template model assumes that he need keep track of his position only in the *relational pattern*, picking up his 'position in the alphabet' from the symbols he has just produced. The template model takes no cognizance of the relationship of the letters *within* a symbol pattern. Note that the descriptions for Problems 5, 11 and 14 (Table 3) differ only in the initial values. Simon & Kotovsky differentiate between an updating of the position in the alphabet and the outputting of a letter. There is no such distinction in the template pattern descriptions, where every update is accompanied by an output.

Some differences in pattern induction

It is not surprising that there are as many differences between the two approaches to *constructing* pattern descriptions as there are between the descriptions themselves. Simon & Kotovsky make many passes over the entire problem in order to build up their pattern description piece by piece. For example, cycle length is determined by finding the occurrence of a 'same' or 'next' at some regular interval for the *entire* problem. Then a regular relation is sought for the *entire* remaining problem, etc. The template building procedure, as outlined earlier, works from the left, and moves to the right only far enough to find some disconfirming evidence for its current hypothesis about the correct pattern. The basic difference in these two approaches is the ability of the template procedure to construct the correct pattern before it has seen the entire problem. To put it more precisely, in a problem whose correct description (in template notation) is a cycle of length n, the template model will have constructed the final version of that description after looking at the first $2n$ letters of the problem.

Simon & Kotovsky discuss at some length the short-term memory load imposed by their pattern descriptions for the problems. However, they devote less attention to the memory load imposed by the process whereby those patterns are induced from the problem elements, a process requiring more effort to construct the simpler patterns than to execute the most complex. In the template building approach, there is an equal distribution of effort between construction and execution of pattern descriptions. For example, the length of the template during induction never exceeds the length of the final pattern description.

Performance comparisons

Information processing models can be compared at two levels. At a 'macro' level i.e. mean error rates, average number of trials to success, or average latencies, one can determine whether or not the average performances of the models are significantly different from one another or from human population means. At a 'micro' level, one can compare the sequence of model actions prior to solution. In this section some macro level comparisons are made among the template model, the Simon & Kotovsky model and some aggregate measures on humans.

Table 4. *Amount of effort on Set I problems for humans, Simon & Kotovsky models and template model*

Problem number	Average time* for successful humans (sec.)	Time for S & K model* (sec.)	Effort template model symbols
1	6·0	9	11
2	3·8	28	22
3	24·7	18	45
4	16·8	23	22
5	27·5	35	33
6	37·0	19	45
7	37·8	19	51
8	24·9	23	18
9	18·8	17	18
10	20·9	18	19
11	21·5	35	29
12	49·8	24	22
13	61·7	29	22
14	41·2	36	29
15	48·0	30	23

* From Simon & Kotovsky (1963, p. 543).

One rough measure of the similarity between a process model and human performance is the relative amount of effort expended by the model and by humans solving the same range of problems. Simon & Kotovsky compare the processing time of their model with the average solution time of those subjects who solved each problem. They find a 'modest' positive correlation between human and model effort. Processing time is used by Simon & Kotovsky as a measure of effort, although as they note, it is only a rough guide since there is no reason to presume a correspondence between relative times for elementary information processes in humans and computers. The measure of effort for the template model is simply a count of symbols produced during a trace like the one shown in Fig. 5. This corresponds to the construction and modification of the relational and symbol templates as they are moved across the problems. Processing time is not used as a direct measure of effort, even though it is very highly correlated with the symbol count ($r = 0{\cdot}9$), because the symbol count is more public and machine independent, in that it can be obtained from a hand simulation of the procedure without any need actually to run the programme. Table 4 lists the effort indices for humans, for the Simon & Kotovsky model and for the template model. For both models the (Spearman) rank correlation of the model effort versus human solution time is low, positive and slightly significant (Simon & Kotovsky–human: $r = 0{\cdot}47$; template–human: $r = 0{\cdot}42$). Thus, at the level of aggregate performance, there is no difference in the predictive ability of these two models. They both provide a moderately good prediction of adult human effort.

Developmental tractability

The template model and the Simon & Kotovsky model of human performance on LSC problems represent two models of quite different detailed structure that are, at an aggregate level, empirically indistinguishable. Gregg & Simon (1967) emphasize universality, precision, simplicity, flexibility, as choice criteria for accepting and entertaining theories in lieu of a clear-cut difference in empirical predictions.

It can be argued that on these criteria the template model is somewhat stronger than the Simon–Kotovsky model; however, in this section another criterion for evaluating models of cognitive behaviour will be introduced. The evaluation should rest partially upon *developmental tractability*, i.e. the ease with which the model can be interpreted as both predecessor and successor of other models in a developmental sequence. The orientation adopted by Piaget towards classical epistemological issues is equally valid in evaluating information processing models, i.e. an orientation that attempts to understand each stage of cognitive ability in terms of its developmental precursor. By this criterion that model is best which is most amenable to transformation into a model of a later developmental stage. Ultimately one hopes to model the transformation process itself in information processing form. This is precisely the sort of approach encouraged by Simon (1962) in his speculative comments on an information processing interpretation of Piaget's concept of 'stage'. (In fact, the examples used in that paper were the early formulations of what became the Simon–Kotovsky model.)

Transition between stages

Consider the two kinds of series completion problems discussed thus far. Call the letter series completion Stage II (see Fig. 5) and the object completion series Stage I (see Fig. 2). In what sense can one speak of development in an organism as it goes from the model of Stage I ability to the model of Stage II ability? The answer lies not in the logic of template building, for the models are almost identical in that respect, but rather in the range of relations necessary for success in constructing the template. The Stage I model has the capacity to test for only a single relation between objects: 'same'. Objects are either the same with respect to an attribute or they are not the same. The Stage II model requires an expanded notion of how objects can be not the same, i.e. a greater range of relations is required. The understanding that 'K' comes after 'B' in the alphabet is a relational concept that differs from the simpler ability to sequentially move through a list of symbols by applying the 'next' operator. It is closely tied to other relational concepts such as 'more' or 'less'. A full understanding of these relationships is clearly absent in 3- to 5-year-old children, for whom both 'more' or 'less' seem to mean 'more' (Donaldson & Balfour, 1968). Such an understanding is unnecessary in the Stage I problem.

Another difference can be found in the complexity of the template building process. The Stage II model requires a second level template of relations that is absent from the Stage I model. Having constructed the Stage II model, one can make it behave like the Stage I model by permitting it to work only with 'sames', i.e. one can simulate Stage I by disabling some of the mechanisms in Stage II. The existence of Stage I within Stage II is a concrete manifestation of the Piagetian concept that early developmental structures should coexist within the framework of later, more complex structures.

A study of developmental tractability

Before one can construct a model of the Stage I to Stage II transition one needs to know something of the behaviour of humans during this transition. This section analyses the problem-solving behaviour of a 5-year-old child during what may be his first encounter with Stage II type problems. Prior to the presentation of these problems this child has been exposed to over 300 colour-orientation problems of the type in Fig. 1.

The problems, Set III (Table 4), are number series completion problems of the same structure as the letter series completion problems in Set I (Table 3). If, in the pattern descriptions of the Set I problems, we substitute a list of the first 26 integers for the English alphabet, then the pattern descriptions for Set I and Set III are the same. (Integers rather than letters are used because children of this age are more familiar with the integers as an ordered list than with the alphabet. The potential attribution of complex arithmetic relations to number problems, a complication not likely in letter problems, does not in fact occur for this subject. Except for one comment about odd and even numbers, there is no mention of any numerical property.)

Table 5. *Problem Set III: number series completion problems*

Number	Problem											
1	3	4	3	4	3	4						
2	1	1	1	2	2	2	3	3	3	4	4	
3	1	7	1	1	7	2	1	7	1	1	7	2
4	1	2	11	3	4	11	5	6	11			
5	4	5	6	7	5	6	7	8	6	7	8	9
6	9	15	1	8	15	2	9	15	1			
7	1	4	13	1	3	13	1	5	13	1	2	13
8	12	1	2	12	2	3	12	3	4	12		
9	6	3	5	6	4	5	6	5	5	6		
10	1	2	15	1	2	14	1	2	13	1	2	
11	14	15	3	4	15	16	4	5	16	17	5	6
12	7	9	1	8	10	1	9	11	1	10	12	1
13	omitted											
14	6	7	14	15	7	8	15	16	8	9	16	17
15	9	8	7	8	7	6	7	6	5	6	5	4

The problems were presented to the subject in the order shown in Table 5. No training series was used. As each problem was presented the subject was asked to predict 'what comes next' and then to explain how he did it. The protocols below have all been generated after the subject has announced his first solution to each problem, after about 20 sec. of silent processing. In terms of the series completion models, the protocols tend to be mixed reports of both the final pattern descriptions and traces of the induction process, although in many cases there appears to be a total recapitulation of the induction process itself.

What follows is an attempt to interpret a few of the protocols, in terms of the development from Stage I to Stage II, of both the model proposed in this paper and that of Simon & Kotovsky. (The entire protocol, with its analysis, is available from the authors upon request.)

Problem 4. 1 2 11 3 4 11 5 6 11

'7. Can you tell me why you are so sure about that? *Well because it goes 1, 2, 11 then 3, 4, 11; 5, 6, 11 then it'd be 7, 8, 11; 9, 10, 11 and so on.* How do you know what the next two are?—*Well it goes 1, 2, 3, 4,—missing out the 11's 5, 6 and then it'd be 7, 8—then 11 again 9, 10 and over again.*'

Here is a comment that is consistent with the Simon–Kotovsky approach. The subject says 'it goes 1, 2, 3, 4—*missing out the 11's*— 5, 6 and then it will be 7, 8 and 11 again...' (our emphasis). Here the cycle length marker, '11', seems to be quite distinct from the series 1, 2, 3, ... The series is compelling in its own right in that it is identical to the external reference alphabet. The two factors probably contribute to the subject's first success in a problem involving both 'sames' and 'nexts' intermixed.

Problem 6. 9 15 1 8 15 2 9 15 1

'Why an 8 next? *Well, 9, 15, 1—[1,2,3] 8, 15—2, 9, 15 then 1 then 8 then 2, 9, 15 then 1, 8, 15 and so on.* So it goes 1, 8, 15—how did you manage to find where that began? That it begins here [1] because it starts with a 9, how did you work it out in your head that it began 1, 8, 15 and so on? *Well 9, 15 and then it goes 9, 15* That's 9, 15 the first two and 9, 15 the last but three and the last but two. *One* [3] *One, [last term] and then it should be 8.* I see 8— the 4th term and 8 next on. *And 15 [5th term] next and then 2 and then the 9 and then 15 so on.*'

This is a trace of the subject's application of the template strategy. The template, constructed from the first six members of the series, resides in the problem itself and is so referred to

Problem 8. 12 1 2 12 2 3 12 3 4 12

'Why 5 next, how does it fit in? *Well it goes 12..1..2—12..2..3—12..3..4, and then it'll be 12..5..6—12..7..8—12..9..10 and so on.* How did you spot that there was 12 and then two numbers? *What do you mean?* When you looked at it carefully you would pick out 12 and two numbers, 12 and two numbers,...how did you work out which number came there after that 12? *Well it goes 12..1..2 then 12..2..3.. 12..4 err 12..3..4, 12, 5, 6.:12, 7, 8..12..9..10 and so on.* These two numbers that are the second part of the group do they go in order all the way along? *Yes.* Read them out, what order do they make? *1, 2; 2, 3; 3, 4; then it'd be 5, 6; 6, 7; 8, 9, 9, 10; 11, 12; 12, 13; and so on.*'

Here the basic cycle, marked by 12's, is clearly acquired by the subject. Then with the experimenter's assistance, a Simon–Kotovsky-like elimination of the 12's is used to detect the relation among the variable elements. In all, the subject goes through three repetitions and extrapolations of the sequence; ignoring the 12s they are: I. 1–2, 2–3. 3–4, *5–6, 7–8, 9–10* . . ; II 1–2, 2–3, 4 . . *3–4, 5–6, 7–8, 9–10* . . ; III. 1–2, 2–3, 3–4, *5–6, 6–7, 8–9, 9–10, 11–12, 12–13* . . It is clear from the last extrapolation that the subject is aware that the variables should repeat as the cycle repeats, but he consistently fails to do it at the discontinuity between problem and extrapolation, and occasionally errs during the extrapolation itself. The description of this failure in our terms is that the correct pattern of *nns* is occasionally misapplied as *lls*. In Simon–Kotovsky terms the subject is unable to do a 'produce' without also doing a 'next'.

Problem 14. 6 7 14 15 7 8 15 16 8 9 16 17

'*9!* How did you think that 9 goes in there? *Well it starts 6, 7, 14, 15, 7, 8. 15, 16, 8, 9, 16, 17.* Then ? *It can't be 9—I think it's 10 then 17, 18.* You mean 9, 10, 17, 18? *Yes.* And what would come after that? After the 18? *11, 12, 18, 19 I think anyway.* How did you work it out from these, how do they go? *6, 7, 14, 15, 7, 8, 15—it goes. I'll do these ones after.* You'll do the second digit in each group after. *6, 7; 7, 8; 8, 9, and then it'd be 9, 10.* And then? *Ah it's gone out of my head 11, 12. No, 10, 11, is that right* I think so. How about these others? *14, 15 then 15, 16 then 16, 17 then 18 no 17, 18 then 19, 20 then 20, 21 then 21, 22 and so on.*'

This is the best example of a Simon–Kotovsky two-position strategy. Haltingly but correctly, the subject is able to overcome the difficulty he had with repeated symbols in problem 8. Perhaps during these 15 or 20 minutes the subject has acquired the facility to move through a list in *other* than a simple series of 'nexts'. Furthermore he has by now acquired the ability to treat two positions on the integer alphabet independently, although not simultaneously.

Problem 15. 9 8 7 8 7 6 7 6 5 6 5 4

'*4 and 3 come next.* How does that fit in altogether? *Well because it goes 9, 8, 7, 8, 7, 6, 7.* Take your time. *9—there's only one 9 right?—and then it goes three 7's and then three 5's* [*counts*]. Is it three 5's? *Oh! I forgot, Oh dear it was going to be 5, 3.* Why is that? *It's one 9, three 7's two 5's and one makes three.* Three 5's! *I don't know whether it's odd or even—odd I think, is it? These: 5, 7, 9?* That's odd yes. *Oh good.* Is there an easier way to put them together? *Well it goes 5, 7, 9 then.* You're pointing out —two [5's] two [7's] and a 9 going back the way. *Oh but it's going to be three [5's] three [7's] and one 9 then a 5 and a 3.* How would a 4 next fit in? *It wouldn't fit in 'cause it'd be one* [*9*] *three* [*7's*] *two* [*5's*] *and one makes 3.* How does that read from the beginning? *9, 8. 7, 8, 7, 6, 7, 6, 6, 6, 5, 4 it'd be 5, 3, 5, 2, 5, 1, 5, 0.*'

The subject is unable to detect the cycle length, probably because there are no fixed elements in the triplet, and all relations, within and between triplets, are 'priors'. Having failed that he turns first to an *extensive* property of the elements in the problem treated as an unordered set of odd and even integers and attempts his 'fairness' strategy of old. After noticing that the last part of the problem is consistent with a cycle length of two, he then extrapolates a pattern based upon *ps* (6, 5) His attempt to construct solutions consistent with only the last part of the problem is similar to behaviour of subjects in binary choice experiments

This mixture of strategies indicates that the issue is not 'which model is correct but rather 'what are the conditions that lead to the adoption of one or the other approach'. This issue is related to the developmental one in that the template building approach, although logically sufficient to solve more advanced problems, seems to be *less* often invoked as the problem complexity increases. This is true even though none of several variants of the Simon–Kotovsky model are sufficiently strong to solve all of the problems in Set III, and even though the template strategy imposes less of a load on short-term memory than the Simon–Kotovsky 'feature scanning' strategy. As the subject becomes familiar with these problems he appears to attempt an embrace of the entire problem at once, and in so doing he abandons the myopia implicit in the successful template strategy.

Progress in this complex area has only begun, but information processing analysis may provide the tools equal to the difficulty of the undertaking.

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